MA10210: ALGEBRA 1B

http://people.bath.ac.uk/aik22/ma10210

Comments on Sheet 1

- Be careful with calculations
  - Often with matrices it is possible to check the results you get:
  - If your answer for $\mathbf{x}$ doesn't solve $\boldsymbol{A}\boldsymbol{x}=\boldsymbol{b}$ you have a problem! ☺ Try to find the problem by looking at how far back in your calculation your solution works.
    (Is it right for the final step? Halfway through?)

Comments on Sheet 1

- Don’t lose information from the matrix
  - To avoid this: don’t take several steps at the same time.
  - If you’re still determined to take several steps at the same time, list the operations in the order you plan to perform them – if operations further down the list involve rows you’re adjusted earlier in the list you have a problem – do these in separate steps.

Warm-up Questions

- Q1
- Q3
- Q5

- Bonus Question:
  - Which of the following form linear subspaces of $\mathbb{R}^3$?

$$x_1 + x_2 + x_3 = 0 \qquad\qquad x_1 x_2 x_3 = 0$$

Bonus Questions

- Find the rank of the following matrices, and the inverses where they exist:

$$\begin{pmatrix} 1 & 2 & 4 & 1 \\ 3 & 3 & 6 & 3 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 3 & -3 & 1 \\ 4 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix}$$

$$\begin{pmatrix} 4 & 7 & 4 \\ 2 & 1 & 4 \\ 1 & 3 & 0 \end{pmatrix}$$

Answers to Bonus Question

- $x_1 + x_2 + x_3 = 0$ - yes

If $\boldsymbol{x} = (x_1, x_2, x_3)$ and $\boldsymbol{y} = (y_1, y_2, y_3)$ are two solutions, then $\boldsymbol{x} + \boldsymbol{y} = (x_1 + y_1, x_2 + y_2, x_3 + y + 3)$ has

$$(x_1 + y_1) + (x_2 + y_2) + (x_3 + y_3) = (x_1 + x_2 + x_3) + (y_1 + y_2 + y_3) = 0$$

so $\boldsymbol{x} + \boldsymbol{y}$ is a solution also.

Hence this is a linear subspace of $\mathbb{R}^n$.

- $x_1 x_2 x_3 = 0$ - no

Consider the solutions
$\boldsymbol{x} = (0, 1, 1)$ and $\boldsymbol{y} = (1, 1, 0)$.
Then $\boldsymbol{x} + \boldsymbol{y} = (1, 2, 1)$ is not a solution.

Bonus Questions (Answers)

- Find the rank of the following matrices, and the inverses where they exist:

$$\begin{pmatrix} 1 & 2 & 4 & 1 \\ 3 & 3 & 6 & 3 \\ 0 & 1 & 2 & 0 \end{pmatrix}$$

rank 2

$$\begin{pmatrix} 3 & -3 & 1 \\ 4 & 1 & 2 \\ 0 & 2 & 0 \end{pmatrix}$$

rank 3

(has inverse)

$$\begin{pmatrix} 4 & 7 & 4 \\ 2 & 1 & 4 \\ 1 & 3 & 0 \end{pmatrix}$$

rank 2

Overview of Sheet 2

- Q2: if you have two solutions to the equation, can you always add them to get a solution?
- Q4: use Q3 and some facts from lectures – make sure you identify what you're using.
- Q6: use elementary row operations – if you manage to invert it, check your answer.
- Q7: to find the null space, solve $\boldsymbol{Ax=0}$.