MA10209 – Week 10 Tutorial

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Top Tips (response to sheet 9)

- Know the difference between elements and sets.
  - When defining a set in terms of its elements, write it as {…}.
- Fibre, Kernel and Image are all sets
  - … and should be written as sets.
- Order of an element is different to order of a group.
  - Similarly know what properties belong to what objects.
    - A set is closed under an operation.
    - An operation is associative on a set.
    - A set contains an identity element, a group has an identity element.
    - Each element has an inverse, a set is closed under taking inverses.

Top Tips (response to sheet 9)

- For H to be a subgroup of G, it must first be a subset.
  - Note: this can include G – so G is a subgroup of G.
  - The elements of H must be identical to elements of G.

- Use sensible names for objects depending on what they are:
  - $a, b, c, \ldots$ elements in sets. Be careful with e, i, j
  - $A, B, C, \ldots$ sets, or if you know it's a group G, H
  - $\alpha, \beta, \gamma, \ldots$ maps
  - Aim to make the notation as easy as possible to follow.
  - This all makes it easier to remember what's what.

Homomorphisms

Let G be a group and let $h \in G$.
Define $\phi : G \to G$ by $g \mapsto hgh^{-1}$.

- Show that ϕ is a group homomorphism.

- Show that $\{hgh^{-1} | g \in G\}$ is a subgroup of *G*.

Cyclic groups

- G is a cyclic group if it can be written as
$$G = \{g^n | n \in \mathbb{Z}\} \text{ for some } g \in G.$$
- Show that every cyclic group is abelian.
- Show that every subgroup of a cyclic group is cyclic.

Exercise Sheet 10 - Overview

- Q1 – working with definition of a subgroups
- Q2 – similar to Sheet 9 Q9, but not the same
- Q3 – two directions
  - use the definition of a homomorphism.
- Q4
  - (b) use the associativity of subset multiplication from (a) (try thinking of aN as AN where A={a})

Exercise Sheet 10 - Overview

- Q5
  - (a) Let **S** be the set of subgroups of G which contain X. Consider $Y = \bigcap_{S \in \mathbf{S}}$.

    show that Y satisfies the conditions for a subgroup.
  - (c) which of the groups you have are subgroups of which?
- Q6 – work with the definition of a subgroup, or the alternative conditions.
  - Bear in mind, if it's not a group then it can't be a subgroup.

Exercise Sheet 10 - Overview

- Q7 – convince yourself of the following:

$$\bigcup_{y\in Y} \bigcup_{z\in Z} yz = \bigcup_{t\in YZ} t$$

- Q8
  - (a) not unlike separation of variables:
    - Take elements of *A* to one side of the equation and elements of *B* to the other. Set these equal to, say, *d*. What do we know about *d*?
- Q9 Show $W \cup \{1\} \leq \langle X \rangle$ and $\langle X \rangle \leq W \cup \{1\}$.