MA10210: ALGEBRA 1B

http://people.bath.ac.uk/aik22/ma10210

(Useful) information

- Hand in sheets by noon on Friday
- Marked sheets given back in the following tutorial

- Tutorial format:
  - Quick comments on previous sheet
    I’m happy to go into more detail on individual questions, but you’ll need to ask
  - Overview of relevant material for next sheet
  - Warm-up (or other) examples
    You have a go, I’ll answer any questions

Row Echelon Form

A matrix $\boldsymbol{A}$ is in *row echelon form* if

- all zero rows are at the bottom,
- the first non-zero entry in each non-zero row is 1, (these are called *pivots*)
- the pivot in row $i + 1$ is strictly right of the pivot in row i.

Reduced Row Echelon Form

A matrix $\boldsymbol{A}$ is in *reduced row echelon form* if

- all zero rows are at the bottom,
- the first non-zero entry in each non-zero row is 1, (these are called *pivots*)
- the pivot in row $i+1$ is strictly right of the pivot in row i,
- all entries above each pivot are 0.

Elementary Row Operations

There are three types of elementary row operation:

$R_i \to \lambda R_i, \quad \lambda \neq 0$	mutiply i^{th} row by λ.
$R_i \to R_i + \lambda R_j$	add a multiple of row to row i.
$R_i \leftrightarrow R_j$	swap rows i and j.

- Write clearly which row operations you're performing.
- Be very careful not to lose a row…

Warm-up Questions

- Q1, Q5
- Bonus Question:

Find the general solution for $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$

where $\boldsymbol{A} = \begin{pmatrix} 2 & 2 & 4 \\ 2 & 4 & 8 \\ 0 & 3 & 6 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 10 \\ 16 \\ 9 \end{pmatrix}$.

Find the solution for $\boldsymbol{A}\boldsymbol{x} = \boldsymbol{b}$, working in the field $\mathbb{F}_3$

where $\boldsymbol{A} = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & -1 \end{pmatrix}$ and $\boldsymbol{b} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$.

Warm-up Questions

- Bonus question solutions:

$$\boldsymbol{x}^{\mathrm{T}} = \begin{pmatrix} 2 & 3 & 0 \end{pmatrix} + \lambda \left(0, -2, 1\right)$$

$$\boldsymbol{x}^{\mathrm{T}} = \left(1, 1, -1\right)$$

Worked solution to bonus question

Reduce the matrix to REF:

$$\left(\begin{array}{ccc|c} 2 & 2 & 4 & 10 \\ 2 & 4 & 8 & 16 \\ 0 & 3 & 6 & 9 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 2 & 4 & 8 & 16 \\ 0 & 3 & 6 & 9 \end{array}\right) \qquad \tfrac{1}{2}R_1$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 2 & 4 & 6 \\ 0 & 3 & 6 & 9 \end{array}\right) \qquad R_2 - 2R_1$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & 2 & 3 \\ 0 & 3 & 6 & 9 \end{array}\right) \qquad \tfrac{1}{2}R_2$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 2 & 5 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad R_3 - 3R_2$$

To further reduce the matrix to RREF:

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 2 \\ 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right) \qquad R_1 - R_2$$

From this point:
No pivot for x_3, so set

$$x_3 = \lambda_1$$

Second row:

$$x_2 + 2x_3 = 3 \Leftrightarrow x_2 = 3 - 2\lambda$$

First row:

$$x_1 = 2$$

So the general solution is:

$$\boldsymbol{x} = \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix} + \lambda \begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}$$

Worked solution to bonus question

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 1 & 1 & -1 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & -1 \end{array}\right) \qquad R_3 - R_1$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right) \qquad R_2 - R_3$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 0 & -1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right) \qquad R_1 - R_3$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 \end{array}\right) \qquad R_1 - R_2$$

So the solution is $\boldsymbol{x} = \begin{pmatrix} 1 \\ 1 \\ -1 \end{pmatrix}$.

Overview of Sheet 1

- Q2: almost identical to warm-up Q1
- Q3: once you put the system into matrix form, it's similar to Q1 & 2
  - a line is $\boldsymbol{r} + \lambda \boldsymbol{s}$
- Q4: $\mathbb{F}_2 = \mathbb{Z}_2$, elementary row operations work as they did before
- Q6: similar feel to Q5 – if your answer needs more than a few lines, there's a neater way to do it
- Q7: similar to example from Lecture 2